

Cosmological meaning of the gravitational gauge group

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Abstract

It is shown that among the $R + \beta S^{abc} S_{abc}$ models, only the one with $\beta = 1/2$ has nonvanishing torsion effect in the Robertson–Walker universe filled with a spin fluid, where S_{abc} denotes torsion. Moreover, the torsion effect in that model is found to be able to replace the big-bang singularity by a big bounce. Furthermore, we find that the model can be obtained under a Kaluza–Klein-like ansatz, by assuming that the gravitational gauge group is the de Sitter group.

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1 Introduction

It has been pointed out that there are three kinds of special relativity (SR) [1–4] and gravity should be based on the localization of a SR with full symmetry [5–7]. It is a motivation for the study of the Poincaré, de Sitter (dS) or Anti-de Sitter (AdS) gauge theory of gravity [8–12], where the Riemann–Cartan (RC) geometry with nontrivial metric and torsion is introduced to realize the corresponding gauge symmetry.

The Kaluza–Klein-like (KK-like) models [12–15] of the gauge theory of gravity are some models in which the Lagrangians are constructed from the scalar curvatures of some fiber bundles, just like the KK theory (for example, see Refs. [16, 17]). The scalar curvature of the bundle is given by the RC geometries of both the spacetime and the typical fiber. In such models, one can see what effects the geometry of the typical fiber, which is determined by the gravitational gauge group, would have on the gravitational dynamics. In other words, one can see what effects the gravitational gauge group would have on the gravitational dynamics.

In the previous KK-like models, it is shown that the fiber geometries produce some quadratic torsion terms [12], or together with some quadratic curvature terms [13, 14], in the gravitational Lagrangian. In these models, however, almost the same dynamics are obtained, when different gauge groups are used. In order to find the effects caused by the difference of the gauge groups, we introduce a coupling constant α between the spacetime and the typical fiber, and construct the corresponding KK-like models in this paper. It is found that the dS group can decide the value of α while the Poincaré group cannot, by the requirement that the no-gravity spacetime is a vacuum solution.

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Moreover, in this paper, it is shown that among the $R + \beta S^{abc} S_{abc}$ models, only the one with $\beta = 1/2$ has nonzero torsion effect in the homogeneous and isotropic universe filled with a spin fluid, where S_{abc} denotes torsion. Also, the torsion effect of the model is found to be able to avert the big-bang singularity of the homogeneous and isotropic universe. On the other hand, somewhat inconceivably, we find that the model is just the aforementioned KK-like model determined by the dS group.

The paper is organized as follows. In section 2, some new KK-like models are constructed by introducing the coupling constant α . In section 3, it is shown that among a class of models, the KK-like model determined by the dS group is the only one that may avert the big-bang singularity under certain assumptions. Finally, we give some remarks in the last section.

2 Kaluza–Klein-like models

Let \mathcal{M} be the spacetime manifold, with the metric g_{ab} , and the metric-compatible derivative operator ∇_a , where a, b are the abstract indices [18, 19]. There exist tensor fields S^c_{ab} and R^c_{dab} , such that for any function f and 1-form field ω_a on \mathcal{M} ,

$$(\nabla_a \nabla_b - \nabla_b \nabla_a)f = -S^c_{ab} \nabla_c f, \quad (1)$$

$$(\nabla_a \nabla_b - \nabla_b \nabla_a)\omega_d = -R^c_{dab} \omega_c - S^c_{ab} \nabla_c \omega_d. \quad (2)$$

The tensor fields S^c_{ab} and R^c_{dab} are called the torsion and curvature tensor fields of ∇_a , respectively. Moreover, the Ricci tensor field and the scalar curvature are defined as $R_{ab} = R^c_{acb}$ and $R = g^{ab} R_{ab}$, respectively. Suppose that $\{e_\alpha^a\}$ is an orthonormal frame field, where $\alpha = 0, 1, 2, 3$. The signature is chosen such that the metric components in $\{e_\alpha^a\}$ are equal to $\eta_{\alpha\beta} = \text{diag}(-1, 1, 1, 1)$. The connection 1-form fields of ∇_a in $\{e_\alpha^a\}$ are defined as

$$\Gamma^\alpha_{\beta a} = e^\alpha_b \nabla_a e^\beta_b, \quad (3)$$

where $\{e^\alpha_b\}$ is the dual frame field of $\{e_\alpha^a\}$. The torsion and curvature 2-form fields of ∇_a in $\{e_\alpha^a\}$ are defined as $S^\alpha_{ab} = S^c_{ab} e^\alpha_c$ and $R^\alpha_{\beta ab} = R^c_{dab} e^\alpha_c e^\beta_d$, respectively. Then Eqs. (1) and (2) imply

$$S^\alpha_{ab} = d_a e^\alpha_b + \Gamma^\alpha_{\beta a} \wedge e^\beta_b, \quad (4)$$

$$R^\alpha_{\beta ab} = d_a \Gamma^\alpha_{\beta b} + \Gamma^\alpha_{\gamma a} \wedge \Gamma^\gamma_{\beta b}, \quad (5)$$

where d_a is the exterior differentiation operator of \mathcal{M} .

The gauge-invariant expressions for the metric, torsion and curvature tensor fields of the spacetime are as follows [9–12]:

$$g_{ab} = \eta_{AB} (D_a \xi^A) (D_b \xi^B), \quad (6)$$

$$S_{cab} = \mathcal{F}_{ABab} (D_c \xi^A) \xi^B, \quad (7)$$

$$R_{cdab} - (2\epsilon/l^2) g_{a[c} g_{d]b} = \mathcal{F}_{ABab} (D_c \xi^A) (D_d \xi^B), \quad (8)$$

where $S_{cab} = g_{cd} S^d_{ab}$, $R_{cdab} = g_{ce} R^e_{dab}$, $\epsilon = 0$ or 1 , l is a constant with the dimension of length, $A, B = 0, 1, 2, 3, 4$, $\eta_{AB} = \text{diag}(-1, 1, 1, 1, 1)$, $D_a \xi^A = d_a \xi^A + \Omega^A_{Ba} \xi^B$, Ω^A_{Ba} is the Ehresmann connection of a principal bundle \mathcal{P}_G , $\mathcal{F}_{ABab} = \eta_{AC} \mathcal{F}^C_{Bab}$, $\mathcal{F}^C_{Bab} = d_a \Omega^C_{Bb} + \Omega^C_{Da} \wedge \Omega^D_{Bb}$, $\xi^A = \xi^A(x)$ are coordinates of a global section ϕ in a fiber bundle

\mathcal{Q}_F as a subbundle of \mathcal{Q}_{M_5} , both of \mathcal{Q}_F and \mathcal{Q}_{M_5} are associated to \mathcal{P}_G , and the typical fiber M_5 of \mathcal{Q}_{M_5} is the 5-dimensional (5d) Minkowski space. When the structure group G of \mathcal{P}_G is the dS group S_{10} , the typical fiber F of \mathcal{Q}_F is a 4d dS space D_4 with radius l , and $\epsilon = 1$. When the structure group G is the Poincaré group P_{10} , the typical fiber F is the 4d Minkowski space M_4 , and $\epsilon = 0$.

Note that ξ^A can be interpreted as the localized 5d Minkowski coordinates, for the reason that the metric field in F can be expressed as $g_{ab} = \eta_{AB}(d_a \xi^A)(d_b \xi^B)$, where ξ^A are the 5d Minkowski coordinates on F as a hypersurface of M_5 . The localized 5d Minkowski coordinates ξ^A are related to the local inertial coordinates y^μ on the spacetime \mathcal{M} , by $y^\mu = \xi^\mu$ when $G = P_{10}$, or $y^\mu = \sqrt{\sigma} \xi^\mu$ when $G = S_{10}$ and $\xi^4 \neq 0$, where $\sigma = (l/\xi^4)^2$, and $\mu = 0, 1, 2, 3$ [2, 9, 11].

The no-gravity spacetime can be defined as a spacetime where $\mathcal{F}^A{}_{Bab} = 0$. The gauge-invariant expressions (7) and (8) imply that the no-gravity spacetime is locally identical with the typical fiber F . It is natural to require that the no-gravity spacetime is a vacuum solution of the gravitational field equations.

The KK-like ansatz we are going to propose is as follows: the gravitational Lagrangian is given by [12]

$$\mathcal{L}_g = \phi^* \bar{R}, \quad (9)$$

where $*$ denotes a pullback, and \bar{R} is the scalar curvature of \mathcal{Q}_F , which is determined by the bundle metric

$$\bar{g} = g_{\mu\nu} dx^\mu \otimes dx^\nu + \alpha \eta_{AB} \theta^A \otimes \theta^B \quad (10)$$

and the bundle torsion

$$\bar{S} = S^\sigma{}_{\mu\nu} X_\sigma \otimes dx^\mu \otimes dx^\nu, \quad (11)$$

where d is the exterior differentiation operator of \mathcal{Q}_F , \otimes stands for a tensor product, $\alpha \neq 0$ is a dimensionless coupling constant between \mathcal{M} and F , $\theta^A = d\xi^A + \Omega^A{}_{B\mu} \xi^B dx^\mu$, $X_\sigma = \partial_\sigma - \Omega^A{}_{B\sigma} \xi^B \partial_A$, x^μ are coordinates on \mathcal{Q}_F and are induced from a coordinate system $\{x^\mu\}$ on \mathcal{M} , $\partial_\sigma = \partial/\partial x^\sigma$, ξ^A (here differ from $\xi^A(x)$) are coordinates on \mathcal{Q}_F as a subbundle of \mathcal{Q}_{M_5} , and are induced from a 5d Minkowski coordinate system of M_5 , $\partial_A = \partial/\partial \xi^A$, $g_{\mu\nu}$, $S^\sigma{}_{\mu\nu}$ and $\Omega^A{}_{B\mu}$ are components of g_{ab} , $S^c{}_{ab}$ and $\Omega^A{}_{Ba}$ in $\{x^\mu\}$. Notice that $\alpha = 1$ in Ref. [12], while it is to be determined here. After some calculations, it can be checked that

$$\phi^* \bar{R} = R + 4\epsilon\Lambda/\alpha - (1/4)\alpha|S|^2, \quad (12)$$

where $\Lambda = 3/l^2$ is the cosmological constant of D_4 , and $|S|^2 = S^{abc}S_{abc}$. When $G = S_{10}$, $\epsilon = 1$, and, to ensure that the no-gravity spacetime is a vacuum solution, $4\Lambda/\alpha$ should be equal to -2Λ . As a result, $\alpha = -2$, and the gravitational Lagrangian is

$$\mathcal{L}_g = \phi^* \bar{R} = R - 2\Lambda + (1/2)|S|^2. \quad (13)$$

When $G = P_{10}$, $\epsilon = 0$, and so the requirement that the no-gravity spacetime is a vacuum solution gives no constraint on α . Consequently, the gravitational Lagrangian is

$$\mathcal{L}_g = \phi^* \bar{R} = R - (1/4)\alpha|S|^2. \quad (14)$$

It can be seen from Eqs. (13) and (14) that different gravitational gauge groups result in different gravitational dynamics.

The following class of Lagrangians contains both of Eqs. (13) and (14) as special cases:

$$\mathcal{L}_g = R - 2\epsilon\Lambda + \beta|S|^2, \quad (15)$$

where β is a dimensionless parameter. The gravitational field equations of the above models are as follows:

$$R_{ba} - \frac{1}{2}Rg_{ab} + \epsilon\Lambda g_{ab} - 2\beta\nabla_c S_{ab}{}^c - \beta S_{acd}T_b{}^{cd} - \frac{1}{2}\beta|S|^2 g_{ab} + 2\beta S^{cd}{}_a S_{cdb} = \frac{1}{2\kappa}\Sigma_{ab}, \quad (16)$$

$$T^a{}_{bc} - 4\beta S_{[bc]}{}^a = -\frac{1}{\kappa}\tau_{bc}{}^a, \quad (17)$$

where $T^a{}_{bc} = S^a{}_{bc} + 2\delta^a{}_{[b}S_{c]}$, $\delta^a{}_b$ is the Kronecker delta, $S_c = S^b{}_{cb}$, κ is the gravitational coupling constant, $\Sigma_{ab} = g_{bc}(\delta S_m/\delta e^\alpha{}_c)e^\alpha{}_a$ is the canonical energy-momentum tensor, $\tau_{bc}{}^a = (\delta S_m/\delta \Gamma^\alpha{}_{\beta a})e^\alpha{}_b e^\beta{}_c$ is the spin tensor, where $e_{\beta c} = \eta_{\alpha\beta}e^\alpha{}_c$, and S_m is the action of the matter fields. Note that the cases with $\beta = 1, 1/4$ or $-1/2$ are problematic because they can only describe the matter fields with $\tau_b \equiv \tau_{bc}{}^c = 0$, $\tau_{[abc]} = 0$, or $g_{a[b}T_{c]} - \tau_{a[bc]} + \tau_{bca} = 0$, respectively. For the cases with $\beta \neq 1, 1/4, -1/2$, the solution of Eq. (17) is

$$S_{abc} = (8\beta^2 + 2\beta - 1)^{-1}[-2(4\beta - 1)g_{a[b}S_{c]} - (2\beta - 1)(1/\kappa)\tau_{bca} + (4\beta/\kappa)\tau_{a[bc]}], \quad (18)$$

where $S_c = [1/2\kappa(1 - \beta)]\tau_c$. The energy-momentum tensor $T_{ab} = -2\delta S_M/\delta g^{ab}$ is related to Σ_{ab} and $\tau_{bc}{}^a$ by

$$\begin{aligned} T_{ab} &= \Sigma_{ab} + (\nabla_c + S_c)(\tau_{ab}{}^c - \tau_a{}^c{}_b + \tau^c{}_{ba}) \\ &= \Sigma_{(ab)} + 2(\nabla_c + S_c)\tau^c{}_{(ab)}. \end{aligned} \quad (19)$$

Hence, if the spin tensor is equal to zero, then $S^c{}_{ab} = 0$, $\Sigma_{ab} = T_{ab}$, and Eq. (16) reduces to the Einstein field equation when $1/2\kappa = 8\pi$, i.e., $\kappa = 1/16\pi$. Generally, Eq. (16) can be expressed as an Einstein-like equation:

$$\mathring{R}_{ab} - \frac{1}{2}\mathring{R}g_{ab} + \epsilon\Lambda g_{ab} = \frac{1}{2\kappa}(T_{\text{eff}})_{ab}, \quad (20)$$

where $\mathring{R} = g^{ab}\mathring{R}_{ab}$, $\mathring{R}_{ab} = \mathring{R}^c{}_{acb}$, $\mathring{R}^c{}_{dab}$ is the torsion-free curvature tensor, and $(T_{\text{eff}})_{ab}$ is the effective energy-momentum tensor which satisfies

$$\begin{aligned} \frac{1}{2\kappa}(T_{\text{eff}})_{ab} &= \frac{1}{2\kappa}T_{ab} - \frac{1}{\kappa}\left(\frac{1}{2}S_{(a}{}^{cd}\tau_{|cd|b)} + K^d{}_{(b|c|}\tau^c{}_{a)d)} + \frac{1}{2}(1 + 2\beta)S^{cd}{}_{(a}S_{b)cd} \right. \\ &\quad \left. - S_{(ab)}{}^c S_c - \frac{1}{4}(T_{cde}K^{dec} - 2\beta|S|^2)g_{ab} - \frac{1}{4}S_{acd}S_b{}^{cd} - 2\beta S_{cda}S^{[cd]}{}_b, \right. \end{aligned} \quad (21)$$

where $K^d{}_{bc} = (1/2)(S^d{}_{bc} + S_{bc}{}^d + S_{cb}{}^d)$ is the contorsion tensor.

3 Cosmological meaning

Let us assume that the matter fields in the universe can be described by a spin fluid [20, 21] with the energy-momentum tensor and spin tensor being

$$T_{ab} = \rho U_a U_b + p(g_{ab} + U_a U_b), \quad (22)$$

$$\tau_{bc}{}^a = \tau_{bc}U^a, \quad (23)$$

where ρ is the rest energy density, p is the hydrostatic pressure, U^a is the four-velocity of the fluid particles, and τ_{bc} is the spin density 2-form which satisfies $\tau_{bc}U^c = 0$. Substitution of Eq. (23) into Eq. (21) yields

$$\begin{aligned} \frac{1}{2\kappa}(T_{\text{eff}})_{ab} = & \frac{1}{2\kappa}T_{ab} + \frac{1}{\kappa^2}(8\beta^2 + 2\beta - 1)^{-2}[(\beta - \frac{1}{2})(8\beta^2 + 2\beta - 1)\tau_{ac}\tau_b{}^c \\ & + (-4\beta^3 - 3\beta^2 + \frac{1}{4})2s^2U_aU_b + (-6\beta^3 - \frac{1}{2}\beta^2 + \beta - \frac{1}{8})2s^2g_{ab}], \end{aligned} \quad (24)$$

where $s^2 = \tau_{bc}\tau^{bc}/2$ is the spin density squire. Furthermore, the metric field of the universe is supposed to be a Robertson–Walker (RW) metric with the line element

$$ds^2 = -dt^2 + a^2(t)[dr^2 + r^2(d\theta^2 + \sin^2\theta d\varphi^2)], \quad (25)$$

where $a(t) > 0$ is called the scale factor, and t is the cosmic time with $(\partial/\partial t)^a = U^a$. According to Eq. (25), the left-hand side of the Einstein-like equation (20) is diagonal in the coordinate system $\{t, r, \theta, \varphi\}$, then the effective energy-momentum tensor $(T_{\text{eff}})_{ab}$ should be diagonal, and so the term containing $\tau_{ac}\tau_b{}^c$ in Eq. (24) should be diagonal, which implies that $\beta = 1/2$ or $\tau_{bc} = 0$. Note that substitution of $\beta = 1/2$ into Eq. (15) leads to Eq. (13). In other words, under the KK-like ansatz (9), the dS group can choose the only one model that is nontrivial in the RW universe filled with a spin fluid (22)(23), among the $R - 2\epsilon\Lambda + \beta|S|^2$ models, while the Poincaré group cannot.

It should be remarked that the torsion field given by Eqs. (18) and (23) is not homogeneous and isotropic in the usual sense. In other words, the Lie derivatives of the torsion field along the Killing vector fields which represent the homogeneous and isotropic properties are not equal to zero. However, we may still think that the torsion field is homogeneous and isotropic for the reason that it is determined by the spin tensor by Eq. (18), and the spin tensor is homogeneous in the sense that the spin density s is a function of the cosmic time, and isotropic in the sense that the spin orientations given by the spin density 2-form τ_{bc} are random at the cosmic scale.

As a matter of fact, each point of the cosmic space corresponds to a region which is small relative to the cosmic space. If the spin orientations are random in such a region, the spin density 2-form should be equal to zero at that point corresponding to this region. In order to obtain a nonzero spin density 2-form, we assume that the spin orientations are aligned in each of such regions.

For the KK-like model (13), Eq. (24) becomes

$$(T_{\text{eff}})_{ab} = T_{ab} - \frac{1}{2\kappa}s^2U_aU_b - \frac{1}{2\kappa}s^2(g_{ab} + U_aU_b). \quad (26)$$

Substitution of $\kappa = 1/16\pi$ and Eq. (22) into the above equation leads to

$$(T_{\text{eff}})_{ab} = (\rho + \rho_S)U_aU_b + (p + p_S)(g_{ab} + U_aU_b), \quad (27)$$

where $\rho_S = p_S = -8\pi s^2$. The early universe is filled with elementary bosons and fermions. In order to preserve the gauge symmetries, we assume that the spin of any massless boson is not coupled to torsion [8]. Furthermore, the spin orientations of the massive bosons and fermions are assumed to be locally aligned, and so $s = \hbar(n_{mb} + n_f/2)$, where n_{mb} and n_f are

the number densities of the spin-1 massive bosons and fermions, respectively. Moreover, it holds [23] that $\rho = 3p = (\pi^2/30)g_*T^4$, $n_{mb} = [\zeta(3)/\pi^2]g_{mb}T^3$ and $n_f = [3\zeta(3)/4\pi^2]g_fT^3$ when the temperature $T \gtrsim 300 \text{ GeV} = 3.480 \times 10^{15} \text{ K}$, where $\zeta(3) \approx 1.202$ is the Riemann zeta function of 3, $g_* = g_b + (7/8)g_f$, and, g_b , g_{mb} , and g_f are the sums of the numbers of interacting spin states for each type of bosons, spin-1 massive bosons and fermions, respectively. For all the known elementary particles, including the Higgs bosons, $g_b = 28$, $g_{mb} = 9$, and $g_f = 90$ [24]. Substituting the above relations into

$$\dot{\rho} + \dot{\rho}_S + (3\dot{a}/a)(\rho + p + 2\rho_S) = 0, \quad (28)$$

which is the conservation law given by the Einstein-like equation (20) and the RW metric, where \cdot denotes the derivative with respect to t , leads to $\dot{T}/T = -\dot{a}/a$ when $2\rho + 3\rho_S \neq 0$, with the solution

$$a = CT^{-1}, \quad (29)$$

which is the same as the standard cosmology, where C is a positive constant. Furthermore, the Friedmann-like equation obtained from the Einstein-like equation (20) and the RW metric is as follows:

$$(\dot{a}/a)^2 = (8\pi/3)(\rho + \rho_S + \rho_\Lambda), \quad (30)$$

where $\rho_\Lambda = \Lambda/8\pi$ can be neglected in the early universe. Consequently, the minimum of the scale factor is approximately given by $\rho + \rho_S = 0$, with the solution $T = T_b \equiv \sqrt{A/B}$ (in the units with $\hbar = c = G = k = 1$), where $A = (\pi^2/30)g_*$, $B = 2[\zeta(3)g_n]^2/\pi^3$, where $g_n = 2g_{mb} + 3g_f/4$. In the international units,

$$T_b = 0.2270 \times T_p = 3.217 \times 10^{31} \text{ K}, \quad (31)$$

where $T_p = \sqrt{\hbar c^5/Gk^2}$ is the Planck temperature. In conclusion, the torsion effect may replace the big-bang singularity by a big bounce with the temperature (31).

The local alignments of the spin orientations may be caused by the cosmic magnetic fields [22], which become weaker when the universe expands, and so the spin orientations would eventually become random, and the torsion effect would vanish.

Note that there is a similar discussion on the early universe [25], where a different spin fluid [26] is used, which satisfies Eq. (23) and

$$\Sigma_{ab} = W_a U_b + p(g_{ab} + U_a U_b), \quad (32)$$

where W_a is the momentum density 1-form. Utilizing the spin conservation law $(\nabla_c + S_c)\tau_{ab}{}^c = -\Sigma_{[ab]}$ and Eq. (23), one can find that

$$\Sigma_{ab} = \rho U_a U_b + p(g_{ab} + U_a U_b) + 2U_b U^c U^d \overset{\circ}{\nabla}_c \tau_{ad}, \quad (33)$$

where $\rho = -W_a U^a$ is the rest energy density. Substitution of Eqs. (33) and (18) into Eq. (19) leads to

$$\begin{aligned} T_{ab} = & \rho U_a U_b + p(g_{ab} + U_a U_b) + 2\overset{\circ}{\nabla}_c \tau^c{}_{(ab)} + 2U^c U^d U_{(b} \overset{\circ}{\nabla}_{|c|} \tau_{a)d} \\ & + (2\beta + 1)^{-1}(4\beta - 1)^{-1}(1 - 2\beta) \frac{1}{\kappa} (2s^2 U_a U_b + \tau_a{}^c \tau_{bc}). \end{aligned} \quad (34)$$

Substituting the above equation into Eq. (24) yields

$$\begin{aligned} (T_{\text{eff}})_{ab} = & (\rho + \rho_S) U_a U_b + (p + p_S)(g_{ab} + U_a U_b) \\ & + 2\overset{\circ}{\nabla}_c \tau^c{}_{(ab)} + 2U^c U^d U_{(b} \overset{\circ}{\nabla}_{|c|} \tau_{a)d}, \end{aligned} \quad (35)$$

where

$$\rho_S = p_S = \frac{4}{\kappa}(2\beta + 1)^{-2}(4\beta - 1)^{-2}(-6\beta^3 - \frac{1}{2}\beta^2 + \beta - \frac{1}{8})s^2. \quad (36)$$

For the RW universe, the last term of Eq. (35) is equal to zero, and the nonzero components of the term $2\overset{\circ}{\nabla}_c \tau^c_{(ab)}$ in Eq. (35) are:

$$2\overset{\circ}{\nabla}_c \tau^c_{(0i)} = \hat{\nabla}^j \hat{\tau}_{ij} = \hat{\varepsilon}_{ijk} \hat{\nabla}^j (*\hat{\tau})^k \equiv [\text{curl}(*\hat{\tau})]_i, \quad (37)$$

where $\hat{\nabla}_i$ is the \hat{g}_{ij} -compatible torsion-free derivative operator, \hat{g}_{ij} is the confinement of g_{ab} on the cosmic space Σ_t , $\hat{\tau}_{ij}$ is the confinement of τ_{bc} on Σ_t , $\hat{\varepsilon}_{ijk}$ is the Levi-Civita symbol, $(*\hat{\tau})_k = \hat{\tau}^{ij} \hat{\varepsilon}_{ijk}/2$ is the spin density 1-form, and, $i, j, k = 1, 2, 3$ and are raised by the inverse of \hat{g}_{ij} . Since the off-diagonal term of $(T_{\text{eff}})_{ab}$ should be equal to zero, Eq. (37) should be identical to zero, which may be inconsistent with the cosmic-scale randomness assumption on the spin orientations. In Refs. [25, 26], the term $2\overset{\circ}{\nabla}_c \tau^c_{(ab)}$ in Eq. (35) with $\beta = 0$ is discarded by an averaging argument: assume that the microscopic spin orientations are random, then the spin tensor and its derivative vanish after macroscopic averaging. However, to my opinion, the averaging procedure is ambiguous. If we average the spin tensor before substituting it into the gravitational field equations, no torsion effect can survive. If the spin tensor is averaged after being substituted into the gravitational field equations, the gravitational field equations should be averaged at the same time, which is an ill-defined concept in a classical theory of gravity.

4 Remarks

We find that under the KK-like ansatz (9), the dS group can choose the only one model that may avert the big-bang singularity of the RW universe filled with a spin fluid (22)(23) by torsion, among the class of models (15), while the Poincaré group cannot. Moreover, the dS group also presents an explanation for the cosmological constant: it is related to the radius l of the no-gravity spacetime by $\Lambda = 3/l^2$. It seems that a regular, accelerating universe favours the dS group as the gravitational gauge group.

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